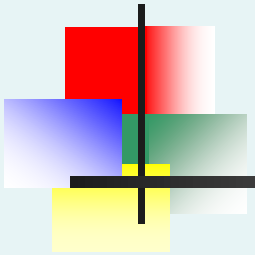
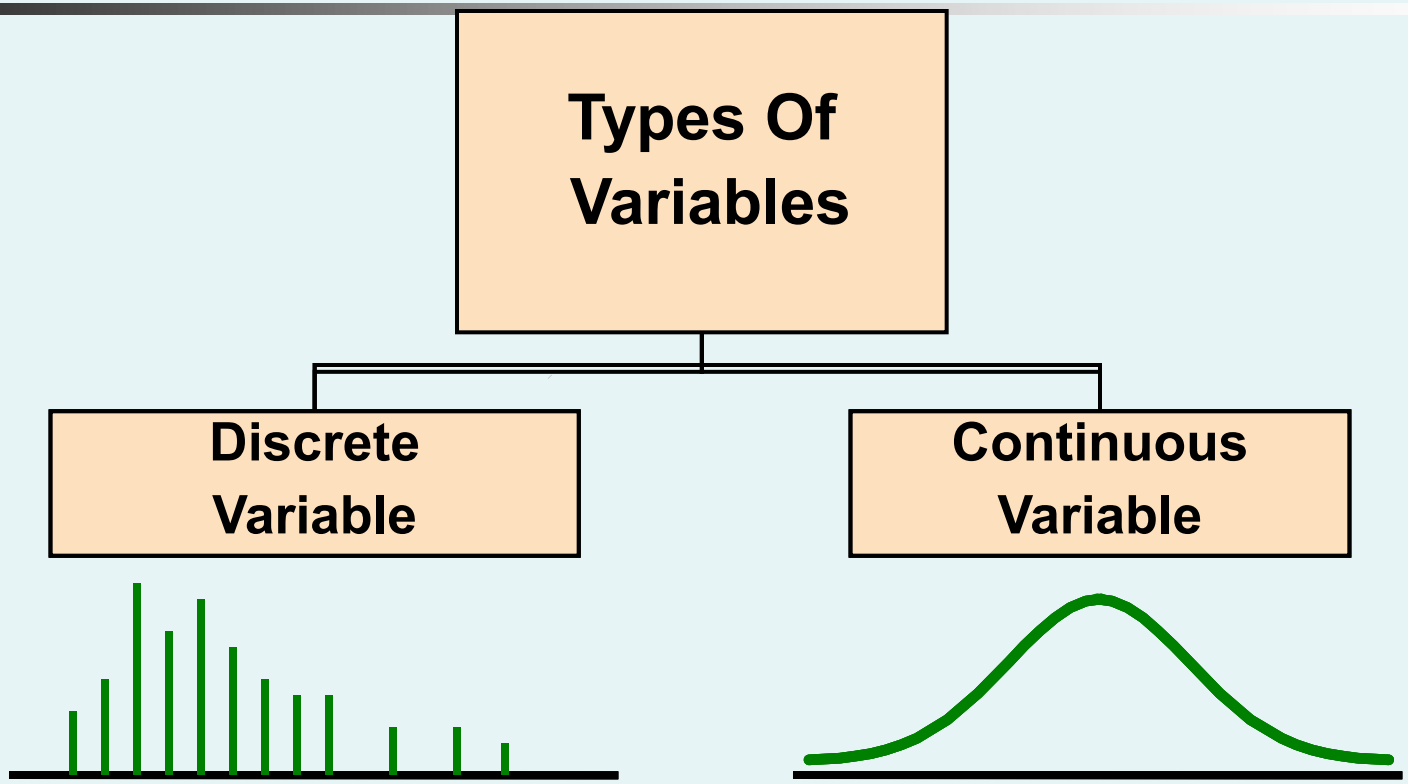
Expected Value and Variance



Types Of Variables



Discrete variables take only separate, isolated values.
Continuous variables can take any value in an interval.

Probability Distribution For A Discrete Random Variable

- *A **probability distribution for a discrete random variable** is a list of its all possible values along with the corresponding probabilities.*

- Example:

x **values of X**	**P(x)** **probabilities**
0	1/8
1	3/8
2	3/8
3	1/8

Expected Value

- Expected Value of a random variable is the mean, or the average:

$$\mu = \mathrm{E}(\mathrm{X}) = \sum_{x} x P(x)$$

- Example: Randomize 2 patients, X = # in the placebo group, compute expected value of X:

E(X) = (0)(0.25) + (1)(0.50) + (2)(0.25)
= 1.0

x	P(x)
0	0.25
1	0.50
2	0.25

Variance and Standard Deviation

- Variance of a discrete random variable measures the spread, or amount of uncertainty, or how far its values can be from the mean:

$$\sigma^2 = Var(X) = E(X-\mu)^2 = \sum_x [x - \mathrm{E}(X)]^2 \mathrm{P}(x)$$

- Standard Deviation of a random variable is the square root. $\sigma = \sqrt{\sigma^2}$ It has the same measurement unit as the original variable *X*.

Covariance

- The covariance measures the strength of the linear relationship between two discrete random variables X and Y.
- A positive covariance indicates a positive relationship.
- A negative covariance indicates a negative relationship.

The Covariance Formula

- The covariance formula:

$$\sigma_{XY} = E(X - \mu_X)(Y - \mu_Y) = \sum_x \sum_y (x - EX)(y - EY)P(x, y)$$

where: X = discrete random variable X
x = value of X
Y = discrete random variable Y
y = value of Y
P(x,y) = P(X=x and Y=y)
= probability of a joint event {X=x and Y=y}

The Sum of Two Random Variables

- Expected Value of the sum of two random variables:

$$E(X+Y)=E(X)+E(Y)$$

- Variance of the sum of two random variables:

$$Var(X+Y)=\sigma^2_{X+Y}=\sigma^2_X+\sigma^2_Y+2\sigma_{XY}$$

- Standard deviation of the sum of two random variables:

$$\sigma_{X+Y}=\sqrt{\sigma^2_{X+Y}}$$

The Weighted Sum Two Random Variables

- Expected Value of a weighted sum:

$$\mathrm{E}(\mathrm{aX}+b\mathrm{Y})=aE(X)+bE(Y)$$

- Variance of a weighted sum:

$$\mathrm{Var}(\mathrm{aX}+b\mathrm{Y})=a^2\sigma_{\mathrm{X}}^2+b^2\sigma_{\mathrm{Y}}^2+2ab\sigma_{XY}$$

- Standard deviation of a weighted sum:

$$\sigma_{aX+bY}=\sqrt{\sigma_{aX+BY}^2}$$